

Modified Entropic Gravitation in Superconductors

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Verlinde recently developed a theoretical account of gravitation in terms of an entropic force. The central element in Verlinde's derivation is information and its relation with entropy through the holographic principle. The application of this approach to the case of superconductors requires to take into account that information associated with superconductor's quantum vacuum energy is not stored on Planck size surface elements, but in four volume cells with Planck-Einstein size. This has profound consequences on the type of gravitational force generated by the quantum vacuum condensate in superconductors, which is closely related with the cosmological repulsive acceleration responsible for the accelerated expansion of the Universe. Remarkably this new gravitational type force depends on the level of breaking of the weak equivalence principle for Cooper pairs in a given superconducting material, which was previously derived by the author starting from similar principles. It is also shown that this new gravitational force can be interpreted as a surface force. The experimental detection of this new repulsive gravitational-type force appears to be challenging.

INTRODUCTION—Recently Verlinde introduced the interesting possibility of gravitation being an entropic force [1]. This approach to the physical nature of gravitation depends strongly on the way in which information is stored in physical systems. The scale at which the information is stored and its location over a surface or a volume are fundamental to determine the type of gravitational force generated by physical systems. Thus the application of Verlinde's procedure to derive the gravitational force produced by a physical system needs to take into account the physical nature of the vacuum associated to its lowest energy level. In the following the electromagnetic zero-point model for the vacuum in superconductors, developed by Beck, Mackey and the author, together with the quantization of information over four-dimensional Planck-Einstein size cells, required by this model, are taken into consideration when applying Verlinde's procedure to deduce the entropic gravitational force generated by the quantum vacuum present in a superconductor. It appears that the quantum vacuum in superconductors generates a gravitational type repulsive force, which is proportional to the superconductor's volume and depends on the level of breaking of the weak equivalence principle for Cooper pairs (for a given superconductor). Following its derivation, The discussion indicates that one can also deduce this force from Sivaram's hydrodynamic procedure to derive Newton gravitational law assuming masses immersed in a cosmic sea of vacuum energy. This exercise shows that one can interpret our basic result, Equ.(29), not only as an entropic force but also in terms of a surface force. The comparison is also made with the cosmological repulsive force, and with a similar force which could arise if we consider that the information of the universe is stored in four dimensional Planck size cells, instead of being stored on Planck size area elements as stipulated by Susskind holographic principle. Although challenging to detect experimentally, in the conclusion one discusses some experimental possibi-

ties to detect the new type of gravitational repulsive force proposed in the present paper, and we close by introducing some theoretical avenues for future work.

INFORMATION THEORY AND UNIVERSAL SCALING RELATIONS FOR THE MASS AND SIZE OF PHYSICAL SYSTEMS—Bekenstein [2] demonstrated from quantum statistical physics and thermodynamics that the ratio of entropy S to mean energy E_0 of a spherical system in its rest frame, is directly proportional to its effective radius R .

$$\frac{S}{E_0} = 2\pi \frac{k}{\hbar c} R \quad (1)$$

Where k is Boltzmann constant, c is the speed of light in vacuum, and \hbar is Planck's constant divided by 2π . The theory of information establishes the general result that the maximum entropy of a given system is directly proportional to the amount of information I , in bits, that the system can store [3].

$$S = \ln(2)kI \quad (2)$$

Substituting Equ.(2) in Equ.(1) one obtains the Bekenstein bound [4], which defines the maximum storage capacity of a system in function of its mean energy E_0 and radius R .

$$I \leq \frac{2\pi}{\ln 2} \frac{1}{\hbar c} E_0 R \quad (3)$$

One can also express this result in function of the system's proper mass $M_0 = E_0/c^2$.

$$I \leq \frac{2\pi}{\ln 2} \frac{c}{\hbar} M_0 R \quad (4)$$

Susskind further proposed the holographic principle, which states that the information contained in a physical system is stored on its boundary, the total number of bits being proportional to the area A , which circumscribes the

system [5].

$$I = \frac{A}{l_p^2} \quad (5)$$

where $l_p = \sqrt{G\hbar/c^3}$ is Planck's length. From the equality between Equ.(4) and Equ.(5) one deduces Sivaram's scaling relation [6] for the mass and size of the Universe, black holes and more generally of objects trapped in their own gravitational field.

$$\frac{M}{R} \sim \frac{c^2}{G} \quad (6)$$

In previous work the author argued that Cooper pairs in superconductors could break the weak equivalence principle on the basis that the information I' associated with the Cooper pair condensate is stored in four dimensional Planck-Einstein size cells [7].

$$I' = \frac{V}{l_{pe}^4} \quad (7)$$

Where V is the spherical four-volume circumscribing the superconductor, and $l_{pe} = (\hbar G/c^3 \Lambda)^{1/4}$ is the Planck-Einstein length, which is a typical scale for dark energy when explained through the cosmological constant Λ [8]. Since all forms of matter and energy in the universe have approximately the same age as the Universe: $T_U = 1/c\Lambda^{1/2}$, the four volume of any physical system is directly proportional to its spatial volume. If we consider a spherical system with radius R we have:

$$V = \frac{4}{3}\pi R^3 c T_U = \frac{4}{3}\pi R^3 \Lambda^{-1/2} \quad (8)$$

Substituting Equ.(8) in Equ.(7) one obtains:

$$I' = \frac{4\pi}{3} \frac{c^3 \Lambda^{1/2}}{\hbar G} R^3 \quad (9)$$

From the equality between Bekenstein's bound, Equ.(4), and Equ.(9) one deduces Sivaram's scaling relation [6] for the mass and size of objects with low surface gravity.

$$\frac{M}{R^2} \sim \frac{c^2}{G} \sqrt{\Lambda} \sim 1 \quad (10)$$

The fact that different versions of the holographic principle, Equ.(5) and Equ.(7), lead to meaningful scaling relations for mass and size of physical objects, when compared to the Bekenstein bound, is an interesting result on its own right. It also demonstrates the great generality of Bekenstein's result, Equ.(4), and is encouraging the further use of the four-dimensional "holographic-type" relation, Equ.(9) in the following part of the present work.

CLASSICAL ENTROPIC GRAVITATION— Let us consider a body with mass M_0 circumscribed by a spherical boundary with radius R , which can be larger than the

physical size of the body which it contains, and a probing mass $m_0 \ll M_0$ which is approaching this spherical boundary.

Verlinde derives Newton's gravitational force as an entropic force from the second law of thermodynamics.

$$F \Delta R = T \Delta S \quad (11)$$

Where F is the gravitational force, ΔR is an infinitesimal variation of the radius of the spherical boundary circumscribing M_0 , T is the equilibrium temperature of the spherical boundary, and ΔS is the variation of entropy associated with the variation of the information stored on the spherical boundary around M_0 resulting from m_0 approaching this boundary.

From the Bekenstein relation between the entropy and the energy of a system, Equ.(1), one deduces that the variation of entropy related to the information on the boundary of the system is:

$$\Delta S = 2\pi \frac{k}{\hbar c} \Delta R E_0 \quad (12)$$

Verlinde's considers that this entropy change is caused by the particle of mass $m_0 = E_0/c^2$ approaching a part of the sphere circumscribing the mass M_0 . Thus one obtains:

$$\Delta S = 2\pi k \frac{m_0 c}{\hbar} \Delta R \quad (13)$$

One can understand this entropy variation as the addition of the entropy carried out by the spherical boundary with radius ΔR circumscribing the mass m_0 , to the entropy stored on the spherical boundary circumscribing the mass M_0 .

The entropic approach to gravitation, Equ.(11), requires to have a temperature T in order to have a force. In Verlinde's model this temperature corresponds to the equilibrium temperature of the boundary circumscribing the mass M_0 . This temperature is calculated assuming that the the energy $M_0 c^2$ is equally distributed among the I bits available over the spherical surface $A = 4\pi R^2$.

$$M_0 c^2 = \frac{1}{2} I k T \quad (14)$$

Substituting Equ.(5) in Equ.(14), one obtains:

$$T = \frac{G \hbar}{2\pi k c} \frac{M}{R^2} \quad (15)$$

Substituting Equ.(15), and Equ.(13) in Equ.(11) one obtains Newton's law of gravitation.

$$F = G \frac{M m}{R^2} = 6.67 \times 10^{-11} \frac{M m}{R^2} \quad (16)$$

ENTROPIC GRAVITATION IN SUPERCONDUCTORS— Beck, Mackey, and the author [9] [10] have developed

an electromagnetic model of vacuum energy in superconductors, starting from the assumption that the virtual photons with zero-point energy

$$\epsilon = h\nu/2 \quad (17)$$

(where h is Planck's constant) form a condensate below the superconductor's critical temperature T_c with energy density:

$$\rho^* = \int_0^{\nu_c} \frac{1}{2} h\nu \frac{4\pi}{c^3} \nu^2 d\nu. \quad (18)$$

In Eq(18) the two possible polarization states of the photon are considered, and ν_c is a certain maximum cutoff frequency. This frequency is calculated by assimilating the condensate of virtual photons with energy $\epsilon_c = h\nu_c/2$ with a black body thermal bath of ordinary photons at the superconductor's critical temperature T_c with mean energy $\bar{\epsilon}_c = h\nu_c/e^{h\nu_c/kT_c} - 1$,

$$\frac{1}{2} h\nu_c = \frac{h\nu_c}{e^{\frac{h\nu_c}{kT_c}} - 1}. \quad (19)$$

This condition is equivalent to:

$$h\nu_c = \ln 3 kT_c \quad (20)$$

Substitution of eq.(20) in eq.(18) leads to the law defining the density of the electromagnetic zero-point energy condensate in function of the superconductor's critical temperature, T_c .

$$\rho^* = \frac{\pi \ln^4 3}{2} \frac{k^4}{(ch)^3} T_c^4 \quad (21)$$

A non-vanishing cosmological constant can be interpreted in terms of a non-vanishing vacuum energy density, ρ_0 , associated with a cosmological quantum vacuum field.

$$\rho_0 = \frac{c^4}{8\pi G} \Lambda \sim 10^{-29} g \text{ cm}^{-3} \simeq 3.88 eV/mm^3 \quad (22)$$

where $\Lambda = 1.29 \times 10^{-52} [m^{-2}]$ is the cosmological constant [11].

In [7], the author argued that the Eötvös factor χ quantifying the level of breaking of the weak equivalence principle by Cooper pairs in superconductors, is proportional to the ratio of the density of electromagnetic zero-point energy (21) to the density of cosmological vacuum energy (22).

$$\chi = \frac{\Delta m_i}{m_g} = \frac{3}{2} \frac{\rho^*}{\rho_0} = \frac{3 \ln^4 3}{8\pi} \frac{k^4 G}{c^7 \hbar^3 \Lambda} T_c^4. \quad (23)$$

Where Δm_i is the anomalous Cooper pair inertial mass excess, which has been measured by Tate and Cabrera for the case of superconducting Niobium [12], and m_g is

the Cooper pair's gravitational rest mass, which is assumed to be equal to the Cooper pairs classical bare mass. Remarkably, Equ.(23) connects the five fundamental constants of nature k, G, c, \hbar, Λ with measurable quantities in a superconductor, χ and T_c . We may define a Planck-Einstein temperature scale T_{PE} streamlined with the Planck-Einstein length introduced above through Equ.(7).

$$T_{PE} = \frac{1}{k} \left(\frac{c^7 \hbar^3 \Lambda}{G} \right)^{1/4} = 60.71 K. \quad (24)$$

Equ.(23) can then be written in a more elegant form [10].

$$\chi = \frac{3 \ln^4 3}{8\pi} \left(\frac{T_c}{T_{PE}} \right)^4. \quad (25)$$

We have now all the elements to apply Verlinde's entropic model of gravity to the case of superconductor's electromagnetic zero-point energy condensate: Like in the previous section, one starts considering that a spherical superconductor with mass M_0 and volume V_0 is circumscribed by a spherical boundary of radius R , in the neighborhood of which a probing mass m_0 is moving. The variation of entropy caused by the mass m_0 is the same as previously calculated for classical materials, i.e. Equ.(13). However the information related with the electromagnetic zero-point energy present in the superconductor is equally stored between the I' Planck-Einstein cells, Equ.(7), available over the four-volume $V = \frac{4}{3}\pi R^3 \Lambda^{-1/2}$ surrounding the superconductor.

$$\rho^* V_{sc} = \frac{1}{2} kT \frac{V}{l_{pe}^4} \quad (26)$$

Where ρ^* is the density of electromagnetic zero-point energy in the superconductor, Equ.(21), $V_{sc} = \frac{4}{3}\pi R_{sc}^3$ is the superconductor volume, and T is the equilibrium temperature of the four-volume V (including, of course, its spherical spatial boundary). Substituting Equ.(21), and the Planck-Einstein length $l_{pe} = (\hbar G/c^3 \Lambda)^{1/4}$ in Equ.(26) one obtains for the temperature:

$$T = \frac{(\ln 3)^4}{2} \frac{k^3 G}{\hbar^2 c^6 \Lambda^{1/2}} T_c^4 \left(\frac{R_{sc}}{R} \right)^3 \quad (27)$$

Substituting Equ.(13) and Equ.(27) in Equ.(11) one obtains the gravitational type force F_{zp} produced by the superconductor's electromagnetic zero-point energy condensate.

$$F_{zp} = \frac{(\ln 3)^4}{4\pi} \frac{k^4 G}{\hbar^3 c^5 \Lambda^{1/2}} T_c^4 \left(\frac{R_{sc}}{R} \right)^3 m \quad (28)$$

Substituting the Cooper pair's Eötvös factor χ Equ.(23) in Equ.(28), I get my basic result in a synthetic form.

$$F_{zp} = \frac{1}{3} c^2 \Lambda^{1/2} \frac{m_0 \chi R_{sc}^3}{R^3} \sim 3.4 \times 10^{-10} \frac{m \chi R_{sc}^3}{R^3} \quad (29)$$

For the case of Niobium $\chi \sim 9.35 \times 10^{-5}$, substituting this value in Equ.(29) and dividing by m_0 one gets the gravitational acceleration generated by the quantum vacuum in superconducting Niobium:

$$a_{zp} \sim 3.18 \times 10^{-14} \frac{R_{sc}^3}{R^3} \quad (30)$$

The dependence on the inverse of the cube of the distance with respect to the superconductor, indicates that the zero-point gravitational type force, Equ.(29) decreases faster than the Newtonian gravitational force which is proportional to the inverse of the square of the distance from the source. Thus at large distances from the superconductor's center of mass, Newtonian gravity will largely dominate over the gravitational zero point force Equ.(29).

DISCUSSION— Comparing the gravitational zero-point force of a superconductor, Equ.(29), with the cosmological repulsive force[13], F_c , responsible for the observed accelerated expansion of the Universe.

$$F_c = \frac{1}{3}c^2\Lambda mr \quad (31)$$

one sees that both forces have similar expressions, and that both should be repulsive forces since latest cosmological observations reveal that the cosmological constant is positive [11], $\Lambda > 0$.

The total gravitational force produced by a superconductor should be understood has being the sum of the classical Newtonian attractive gravitational force originating from the total gravitational mass of the superconductor M_{sc} , and the repulsive zero-point gravitational force originating from the Electromagnetic zero-point condensate in the superconductor.

$$\vec{F}_{grav} = \frac{m}{R^2} \left(\frac{1}{3}c^2\Lambda^{1/2} \frac{\chi R_{sc}^3}{R} - GM_{sc} \right) \hat{r} \quad (32)$$

where \hat{r} is the radial unit vector pointing outwards with respect to the superconductor center of mass.

In the context of the results just derived, it is instructive to compare Verlinde's procedure to derive the Newtonian gravitational force with Sivaram's derivation, which achieves the same result on the basis of the laws of hydrodynamics [14]. It is impressive to see that by applying Sivaram's procedure to the case of the electromagnetic zero-point model of vacuum in superconductors, introduced above, one succeeds to find back the same repulsive zero-point gravitational force obtained in Equ.(29).

Sivaram starts by considering that a mass M_0 , circumscribed by a spherical shell with radius R , will affect the density of the vacuum energy due the the spacetime curvature it generates.

$$P'_{vac} = P_{vac} \left(1 - \frac{2GM_0}{Rc^2} \right) \quad (33)$$

where P'_{vac} vacuum pressure (or identically, the vacuum energy density) inside the shell, $P_{vac} = \rho_0 = c^4\Lambda/8\pi G$ is the ambient vacuum pressure outside the shell. Thus the pressure difference across the shell will be.

$$\Delta P_{vac} = P_{vac} - P'_{vac} = P_{vac} \frac{GM}{c^2 R} \quad (34)$$

Inspired by the Archimedes principle, Sivaram proposed that any object of mass m_0 in the vacuum fluid displaces a volume V , such that:

$$m_0 c^2 = P_{vac} V \quad (35)$$

Multiplying both sides of Equ.(34) by the displaced volume of vacuum fluid V , one obtains an expression analogous to the one giving the buoyant Archimedes force.

$$\Delta P_{vac} V = P_{vac} V \frac{GM}{c^2 R} \quad (36)$$

Substituting Equ.(35) in Equ.(36) one obtains the Newtonian gravitational potential energy, which is binding together the masses M_0 and m_0 .

$$U_g = \frac{GM_0 m_0}{R} \quad (37)$$

Taking the gradient of Equ.(37), i.e. the pressure gradient of the vacuum energy, one obtains Newton's law.

$$\vec{F} = -G \frac{M_0 m_0}{R^2} \hat{R} \quad (38)$$

Thus we have a hydrodynamics derivation of Newton's law indicating that one can interpret Newton's gravitational force as a Buoyant type force between masses immersed in a sea of vacuum energy.

To apply this procedure to the case of a superconductor one must take into account that the vacuum energy density inside a superconductor ρ^* is given by Equ.(21).

$$\rho^* = \frac{2}{3}\chi\rho_0 = \frac{2}{3}\chi P_{vac} \quad (39)$$

If one imagines a spherical membrane with radius R circumscribing the spherical superconductor of radius R_{sc} , the vacuum pressure inside the membrane P'_{vac} is:

$$P'_{vac} = P_{vac} - \rho^* \quad (40)$$

Substituting Equ.(39) into Equ.(40) one obtains the pressure difference across the membrane.

$$\Delta P_{vac} = \frac{2}{3}\chi P_{vac} \quad (41)$$

Setting the total vacuum energy in the superconductor, $\frac{4}{3}\pi R_{sc}^3 \rho^*$, equal to the total difference of vacuum energy across the membrane of radius R .

$$\Delta P_{vac} \frac{4}{3}\pi R^3 = \frac{2}{3}\chi P_{vac} \frac{4}{3}\pi R_{sc}^3 \quad (42)$$

One obtains the pressure difference across the membrane in function of the inverse of the cube of the distance from the superconductor.

$$\Delta P_{vac} = \frac{2}{3} \chi P_{vac} \left(\frac{R_{sc}}{R} \right)^3 \quad (43)$$

Sivaram [6] scaling relation Equ.(10) also implies a universal relation for the surface tension T (expressed in *Joules/Area*) of physical systems, which states that the surface tension of a physical system is always approximately equal to the surface tension of the Universe.

$$T = \rho_0 R_{Universe} = \frac{\rho_0}{\Lambda^{1/2}} = \frac{P_{vac}}{\Lambda^{1/2}} \quad (44)$$

Substituting Equ.(22) into Equ.(44) one gets:

$$T \sim \frac{\Lambda^{1/2} c^4}{G} \quad (45)$$

Applying Sivaram's universal scaling relation for tension to the case of a probing mass m_0 moving in the neighborhood of the mass M_0 , one can calculate the cross sectional area σ of the moving particle m_0 , which defines the gravitational surface interaction between the two masses. This is achieved by substituting the tension $T = m_0 c^2 / \sigma$ in Equ.(44).

$$\frac{m c^2}{\sigma} = \frac{P_{vac}}{\Lambda^{1/2}} \quad (46)$$

Multiplying both sides of Equ.(43) by the area σ calculated from Equ.(46), one obtains a surface gravitational type force very close to the zero point gravitational type force, Equ.(29), derived from Verlinde's procedure applied to superconductors.

$$F_s = \frac{2}{3} c^2 \Lambda^{1/2} \frac{m_0 \chi R_{sc}}{R^3} \quad (47)$$

Note that since $P_{vac} < P'_{vac}$ this force is repulsive with respect to the superconductor since it will always point from high to low vacuum pressure. Thus one should also observe a tiny negative pressure on the superconductor [7]. The important lesson one learns from this derivation is that the repulsive zero-point gravitational type force produced by superconductors can be interpreted not only in terms of an entropic force but also as a surface-type force.

Before one closes the discussion, let us deduce the consequences of Sorkin's alternative to the holographic principle to account for the small value of the cosmological constant [8]. Sorkin's suggested that the total amount of information stored in the Universe is directly proportional to the universe four-volume [15][16]:

$$I'' = \frac{V}{l_p^4} \quad (48)$$

Where $V = \frac{4}{3} \pi R^3 c^{-1} \Lambda^{-1/2}$ is the four volume of the universe and $l_p = \sqrt{G \hbar / c^3}$ is Planck's length. From the equality between Equ.(48) and the Bekenstein bound Equ.(4) one obtains a new scaling relation for mass and length of physical bodies:

$$\frac{M}{R^2} = \frac{2}{3} \ln 2 \frac{c^5}{c^2 \hbar \Lambda^{1/2}} \quad (49)$$

Calculating the equilibrium temperature of the universe spherical boundary following Verlinde's procedure, already used several times above,

$$M_0 c^2 = \frac{1}{2} k T I'' \quad (50)$$

and substituting Equ.(48) in Equ.(50) one obtains for the equilibrium temperature:

$$T = \frac{3}{2\pi} \frac{\Lambda^{1/2} G^2 \hbar^2}{k c^4} \frac{M}{R^3} \quad (51)$$

Substituting Equ.(51) and Equ.(13) in Equ.(11) one obtains the gravitational force law:

$$F = 3 \frac{\Lambda^{1/2} G^2 \hbar}{c^3} \frac{M_0 m_0}{R^3} \sim 5.93 \times 10^{-106} \frac{M_0 m_0}{R^3} \quad (52)$$

Comparing relations Equ.(49) and Equ.(52) with the equivalent relations Equ.(6), Equ.(16) and Equ.(10), Equ.(29) respectively, one sees that relations Equ.(49) and Equ.(52) cannot be valid for physical systems inside our own Universe. One could speculatively propose that the scaling relation Equ.(49) could be understood as holding for possible different universes, and that the force law Equ.(52) could be related with the gravitational force between different universes of total mass m_0 and M_0 respectively. This also contributes to indicate that our basic result Equ.(29) can only be obtained in the context of the Planck-Einstein scale.

CONCLUSIONS— Combining Verlinde's entropic account of gravitation, Equ.(11), with the electromagnetic zero-point model of quantum vacuum in superconductors, Equ.(21), one deduces a new type of repulsive gravitational force generated by the electromagnetic zero-point vacuum energy condensate contained in the crystal of a superconductor Equ.(29). Comparing this derivation with Sivaram's procedure to obtain Newton's gravitational law from the hydrodynamics of vacuum, applied to superconductors, one deduces that this repulsive zero-point gravitational force is also a surface force. The results and discussion presented in this paper, suggests carrying out Eötvös type experiments with superconducting masses aiming at measuring any anomalous value of the universal gravitational constant, which could be accounted for by the repulsive gravitational force Equ.(29). In addition, one could also suggest repeating the small scale tests of the gravitational inverse square law as performed by Adelberger et al but with superconducting

probing masses, instead of classical masses [8], in order to confirm the dependence on the inverse of the cube of the distance of the new gravitational repulsive law Equ.(29). On the theoretical side it would be interesting to explore any possible relationship between the surface force derived in the present work, Equ.(29), with the Tao surface force in superconducting millimetric balls [17], and with the surface force predicted by Ulf Leonhardt for the optical analogue of the Iordanskii force in rotating Bose-Einstein condensates [18].

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